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# Non-commutative holographic QCD and DC conductivity

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**Abstract**

In this paper we consider non-commutative Sakai-Sugimoto model [5] by using D8-branes probing the background generated by D4-branes with an NSNS B-field turned on. We study response of the system to external electric field which could be parallel to orthogonal to the background B-field. We compute the conductivity as a function of temperature and non-commutativity parameters. Non-commutativity effect, depending on the relative orientation of external and background electromagnetic fields, may increase or decrease the conductivity.

# 1 Introduction

According to the Anti-de Sitter/conformal field theory (AdS/CFT) conjecture, IIB string theory on $AdS_5 \times S^5$ background is dual to $D = 4$ $\mathcal{N} = 4$ $SU(N_c)$ super Yang-Mills theory (SYM) [1]. In the large $N_c$ and large 't Hooft coupling $\lambda = g_{YM}^2 N_c$ limit, the SYM theory is dual to IIb supergravity which is low energy effective theory of superstring theory. As a result a strongly coupled thermal SYM theory corresponds to supergravity in an AdS black brane background where the SYM theory temperature is identified with the Hawking temperature of the AdS black hole [2].

AdS/CFT idea has been applied to study different aspects of strongly coupled gauge theories. Recently the application of this duality in condensed matter physics (called AdS/CMT) has been studied [3] . This duality is very useful to study certain strongly coupled systems in CMT by holographic AdS/CFT techniques and to understand better their properties. Of interest is to compute response of the system to the external forces or fields, in particular the response to the external electric field, the conductivity [4].

There are two important holographic models describing quantum chromodynamics (QCD) which are based on dynamics of D3-D7 or D4-D8 (the Sakai-Sogimoto model) [5, 6] systems. Various different aspects of these models such as phase diagram, chiral symmetry breaking, hadronic spectrum and response to the external electric-magnetic fields have been extensively studied in these contexts in the literature [4, 7, 8, 9, 10]. In order to compute conductivity in D3-D7 system, on the gravity side, $N_f$ flavor D7-branes are introduced in the probe limit, *i.e.* $N_f \ll N_c$ and hence AdS background is left unchanged. On the gauge theory side introduction of $N_f$ D7-branes amounts to introducing $\mathcal{N}$=2 hypermultiplets in the fundamental representation of the gauge group. These hypermultiplets which are in $U(N_f)$ representation may be associated with the open strings between the D7 flavor branes and the D3-branes of the background. The local $U(N_f)$ symmetry on the D7-branes corresponds to global $U(N_f)$ symmetry whose $U(1)_B$ subgroup may be identified with baryon number. Non-dynamical electric and magnetic fields can be coupled to $U(1)_B$ charge and we then expect a constant, nonzero current. Hence the conductivity tensor $\sigma_{ij}$ is identified by

$$< J_i >= \sigma_{ij} E_j \,. \tag{1}$$

Electric and magnetic fields produce diagonal and off-diagonal elements in conductivity tensor respectively. In fact on the gravity side currents, electric

and magnetic fields are introduced as nontrivial gauge fields living on the D7-branes [8].

In D4-D8 system, D8-branes have the same role as D7-branes in pervious case and hence open strings stretched between the $N_f$ flavor D8-branes and $N_c$ color D4-branes are associated with fields in fundamental representation in the CFT side. However D3-D7 system is not a chiral model, D4-D8 model enjoys chiral symmetry and explains chiral symmetry breaking [5, 6]. Response of the system to the external electromagnetic field and conductivity in D4-D8 system were also discussed in [9].

Non-commutative gauge theories naturally appear on the D-branes with a background NSNS B-field on them. Explicitly consider a system of D$p$-branes with a constant NSNS B-field along their worldvolume directions. By taking a low energy limit , closed strings decouple and the resulting action for open strings is the non-commutative gauge theory [11]. It is possible to extend the AdS/CFT dictionary to the cases involving background B-field in the gravity side and non-commutative gauge theory in the CFT side [12, 13].

What we will consider in this paper is the effect of the NSNS B-field on the conductivity in the Sakai-Sugimoto model which will be reviewed in section 2. In section 3, a number of D8-branes are embedded in the "non-commutative geometry" corresponding to the non-commutative QCD, hence building a non-commutative Sakai-Sugimoto model. We then find conductivity in non-commutative gauge theories at low and high temperatures in this non-commutative Sakai-Sugimoto model. In next section we consider a more general non-commutative Sakai-Sugimoto background in which two independent components of NS B-field $(B_{01}, B_{23})$ are turned on and study the conductivity. The last section is devoted to conclusion.

# 2 A brief review on Sakai-Sugimoto model

In this section, we review holographic QCD background (Sakai-Sugimoto model) at low and high temperature [5, 6]. The holographic QCD model is constructed from the near horizon limit of a set of $N_c$ D4-branes compactified on a circle with an anti-periodic boundary condition for the adjoint fermions. This makes the adjoint fermions massive and breaks supersymmetry. Fermions in (anti-)fundamental representation are introduced by $N_f$ (anti-)D8-branes intersecting the D4-brane at a 3+1 -dimensional defect. There is thus a global $U(N_f) \times U(N_f)$ chiral symmetry from the worldvol-

ume of D4-brane point of view. We work in the prob limit, namely $N_f \ll N_c$, where flavour branes do not backreact on the background.

At low temperature, the near horizon of D4-branes reads [6]

$$\begin{aligned} ds^2 &= (\frac{u}{R})^{3/2}\left(dt_E^2 + dx^i dx_i + f(u)dx_4^2\right) + (\frac{R}{u})^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \\ e^\phi &= g_s(\frac{u}{R})^{3/4}, \qquad f(u) = 1 - \frac{u_k^3}{u^3}, \qquad F_4 = dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \end{aligned} \tag{2}$$

where $t_E$ (Euclidean time), $x^i (i = 1, 2, 3)$ and $x_4$ are the directions along which the D4-branes are extended. $d\Omega_4^2, \epsilon_4$ and $V_4 = 8\pi^2/3$ are the line element, the volume form and the volume of a unit $S^4$, respectively. $R$ and $u_k$ are constant parameters. $R$ is related to the string coupling $g_s$ and string length $l_s$ as $R^3 = \pi g_s N_c l_s^3$. The coordinate $u$ is bounded from below by the condition $u \geq u_k$. In order to avoid a singularity at $u = u_k$, $x_4$ must be a periodic variable with period $\mathcal{R}$

$$2\pi\mathcal{R} = \frac{4\pi}{3}(\frac{R^3}{u_k})^{1/2} \tag{3}$$

The high temperature background is given by [6]

$$\begin{aligned} ds^2 &= (\frac{u}{R})^{3/2}\left(f(u)dt_E^2 + dx^i dx_i + dx_4^2\right) + (\frac{R}{u})^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \\ f(u) &= 1 - \frac{u_T^3}{u^3}, \qquad \delta t_E = \frac{4\pi}{3}(\frac{R^3}{u_T})^{1/2} = \frac{1}{T} = \beta, \qquad u_T = (\frac{4\pi T}{3})^2, \end{aligned} \tag{4}$$

where $T$ is the background temperature and all the other parameters are defined as before.

It was shown in [6] that this theory undergoes a confinement-deconfinement phase transition at a temperature $T_d = 1/2\pi\mathcal{R}$. For quark separation obeying $R > 0.97\mathcal{R}$, the chiral symmetry is restored at this temperature but for $R < 0.97\mathcal{R}$ there is an intermediate phase which is deconfined with broken chiral symmetry and the chiral symmetry is restored at $T_{\chi SB} = 0.154R$.

# 3 Non-commutative conductivity

The response of the D3-D7 system (dual to $\mathcal{N} = 2$ SYM with $N_f$ flavor hypermultiplets) to external electric field $E$ was originally discussed in [4],

where it was found that in the massless limit the conductivity $\sigma$ is given by

$$\sigma^2 = \frac{N_f^2 N_c^2 T^2}{16\pi^2}\sqrt{e^2+1} + \frac{d^2}{e^2+1} \tag{5}$$

where $N_c$ and $T$ are related to radius and temperature of AdS black hole. Moreover $e$ and $d$ are defined as

$$e = \frac{E}{\frac{\pi}{2}\sqrt{\lambda}T^2}, \qquad d = \frac{D}{\frac{\pi}{2}\sqrt{\lambda}T^2} \tag{6}$$

where $\lambda = g_s^2 N_c$. $E$ and $D$ are external electric field and (free) charge density introduced by non-trivial gauge field living on D7-branes. One can eliminate the effect of free charge in (5) by setting $A_0$ to zero on branes [4]. Hence the first term in (5) in two different limits becomes

- Zero mass, zero density and zero external field

$$\sigma = \frac{N_f N_c}{4\pi} T \tag{7}$$

  The factor $N_f N_c$ is number of charge carriers.

- Zero mass, zero density and zero temperature [1]

$$\sigma^2 = \frac{N_f^2 N_c^2}{8\pi^3}\frac{E}{\sqrt{\lambda}} \tag{8}$$

What we are going to do in the following subsections is to find conductivity in low and high temperature within the non-commutative Sakai-Sugimoto model. To do so, we add a number $N_f$ of D8-branes filling all directions except $x_4$ in non-commutative background. For simplicity $A_0$ is set to zero and therefore the effect of free charge density does not appear in our result.

[1] $\frac{E}{\sqrt{\lambda}}$ is the natural combination which appears in the strong coupled SYM regime *e.g.* [15].

## 3.1 Low temperature case

The low temperature non-commutative background is given by [2] [14]

$$
\begin{aligned}
ds^2 &= u^{3/2}\Big(h(u)(dt_E^2 + dx_1^2) + dx_2^2 + dx_3^2 + f(u)dx_4^2\Big) \\
&\quad + u^{-3/2}\Big(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\Big), \\
h(u) &= \frac{1}{1+\theta^3 u^3}, \qquad e^{\phi} = g_s u^{3/4}\sqrt{h(u)}, \qquad B = B_{t1} = \theta^{3/2}u^3 h(u),
\end{aligned}
\tag{9}
$$

where $\theta$ is non-commutative parameter. This metric reduces to (2) by setting $\theta$ to zero.

The low energy effective action for $N_f$ D8-branes in an arbitrary background is given by Dirac-Born-infeld (DBI) action [3]

$$
S_{D8} = -T_8 N_f \int d^9\xi e^{-\phi}\sqrt{\det\Big(g_{\mu\nu} + B_{\mu\nu} + (2\pi\alpha')F_{\mu\nu}\Big)} \tag{10}
$$

where $\mu$ is worldvolume index running from 0,...,9. $g_{\mu\nu}$ and $B_{\mu\nu}$ are induced metric and induced Kalb-Ramond fields defined by

$$
\begin{aligned}
g_{\mu\nu} &= G_{MN}\partial_\mu X^M \partial_\nu X^N, \\
B_{\mu\nu} &= B_{MN}\partial_\mu X^M \partial_\nu X^N.
\end{aligned}
\tag{11}
$$

In our case $G_{MN}$ is non-commutative metric (9) where $M, N = 0, ..., 9$. $F_{\mu\nu}$ is field strength of the gauge field living on the brane. The brane tension is $T_8 = \frac{2\pi}{(2\pi l_s)^9 g_s}$ and $\phi$ is the dilaton field. Note that in this background Klab-Ramond field is no longer zero and it therefore contributes to DBI action.

Let us start with following ansatz for $A_1(u, t_E)$ in Euclidean space

$$
A_1(t_E, u) = iEt_E + a_1(u). \tag{12}
$$

In static gauge, the DBI action for $N_f$ D8-branes in the low temperature background (9) becomes

$$
S_{D8} = -\frac{8\pi^2}{3}N_f T_8 \int du dt_E \frac{1}{\sqrt{h}}\sqrt{\hat{g}_{22}g_{33}\big[g_{tt}(\hat{a}_1'^2 + g_{11}g_{uu}) - g_{uu}(B^2 + \hat{E}^2)\big]}\ , \tag{13}
$$

[2]Hereafter we consider non-commutative metrics in units where the background $AdS$ radius is one.

[3]Note also that Chern-Simon action does not contribute in (10) for our case.

where $\hat{E} = (2\pi\alpha')E$, $\hat{g}_{22} = u^{1/2}g_{22}$ and $\hat{a}_1 = (2\pi\alpha')a_1$. Since the above action will only depend upon $u$-derivative of $a_1(u)$, we will have a conserved charge associated with $a_1(u)$ which is given by

$$I_1 \equiv \frac{1}{\sqrt{h}} \frac{\mathcal{N}(2\pi\alpha')^2 \hat{g}_{22} g_{tt} g_{33} a_1'}{\sqrt{\hat{g}_{22} g_{33} \big[g_{tt}(\hat{a}_1'^2 + g_{11} g_{uu}) - g_{uu}(B^2 + \hat{E}^2)\big]}}. \tag{14}$$

where [4]

$$\mathcal{N} = \frac{8\pi^2}{3} N_f T_8 = \frac{1}{6(2\pi)^6} N_c N_f \lambda^2. \tag{15}$$

Solving (14) for the gauge field leads to

$$\hat{a}_1' = \pm\sqrt{\frac{h g_{uu}\big(B^2 + \hat{E}^2 - g_{11} g_{tt}\big)}{g_{tt}\big(h I_1^2 - \mathcal{N}^2 (2\pi\alpha')^2 \hat{g}_{22} g_{33} g_{tt}\big)}}\; I_1. \tag{16}$$

According to AdS/CFT dictionary, it was shown in [4] that $I_1$ is a source term for $a_1$ at the boundary of AdS black hole. More precisely the expectation value of $J_1$ on the boundary is considered to be $I_1$. We assume that the standard AdS/CFT formulation and hence the relation between $I_1$ and $J_1$ still exists in non-commutative background. $a_1(u)$ is a real field and the only way for (16) to remain real for all values of $u$ is if both numerator and denominator change sign at the same point *i.e.* $u = u_*$. The reality condition of the gauge field then yields to conductivity equations as

$$g_{11} g_{tt} - (B^2 + \hat{E}^2) = 0, \tag{17a}$$
$$h I_1^2 - \mathcal{N}^2 (2\pi\alpha')^2 \hat{g}_{22} g_{33} g_{tt} = 0, \tag{17b}$$

where all induced metric components are evaluated at $u_*$. At it is seen from (17a), the $B$ field appears as an electric field added to external field $E$. The backreaction of $B$ on the background is already considered. One should check that the backreaction of $E$ is on the background ignorable. This happens if the value of $E$ is smaller than the value of $B$ on the brane *i.e.* $E < B$ which is equivalent to

$$\theta^3 \hat{E}^2 < 1, \tag{18}$$

for large values of $u$. The backreaction of $E$ is thus negligible if it satisfies (18).

[4]Note that by choosing $R = 1$, we have $\lambda = 4\pi g_s N_c \alpha'^{1/2} = \frac{4}{\alpha'}$.

After substituting induced metric components and $B$ in (17a), one obtains

$$u_*^3(1-\theta^3 u_*^3)-(1+\theta^3 u_*^3)^2\hat{E}^2=0, \tag{19}$$

whose roots are

$$u_*^3=\frac{1-2\theta^3\hat{E}^2\pm\sqrt{1-8\theta^3\hat{E}^2}}{2\theta^3(1+\theta^3\hat{E}^2)}\ . \tag{20}$$

where the reality of $u_*$ imposes $\theta^3\hat{E}^2\leq\frac{1}{8}$ which is stronger than (18). We use (17b) and (20) to find conductivity in terms of electric field which is finally given by [5]

$$\sigma_{11}^2=\frac{I_1^2}{E^2}=(2\pi\alpha')^4\mathcal{N}^2\hat{E}^{4/3}(1+5\theta^3\hat{E}^2), \tag{21}$$

up to $O(\theta^6)$. In this case non-commutativity increases the value of conductivity. For the critical maximum possible electric field E for a given $\theta$, $\theta^3\hat{E}^2=1/8$, the conductivity is obtained as

$$\sigma_{11}^2=(2\pi\alpha')^4\mathcal{N}^2\frac{64\theta}{3^{5/3}} \tag{22}$$

### 3.2 High temperature case

The high temperature non-commutative background is given by

$$\begin{aligned} ds^2 =& u^{3/2}\Big(h(u)\Big(f(u)dt_E^2+dx_1^2\Big)+dx_2^2+dx_3^2+dx_4^2\Big)\\ &+u^{-3/2}\Big(\frac{du^2}{f(u)}+u^2d\Omega_4^2\Big), \end{aligned} \tag{23}$$

where all other parameters are the same as (4) and (9). In high temperature, as it was in pervious case, $A_1(u,t)$ is taken to be

$$A_1(t_E,u)=iEt_E+a_1(u). \tag{24}$$

[5]For small value of $\theta$ one can extend (20) yielding

$$u_*^3=\frac{1}{2\theta^3}\Big(1-2\theta^3\hat{E}^2\pm(1-4\theta^3\hat{E}^2)+O(\theta^6)\Big)$$

Obviously minus sign reproduces commutative solution at zero temperature.

Hence the DBI action for $N_f$ D8-branes in the high temperature background is the same as (13). It is easy to see that conductivity equations also become the same as (17)

$$g_{11}g_{tt} - (B^2 + \hat{E}^2) = 0 \tag{25a}$$
$$hI_1^2 - \mathcal{N}^2(2\pi\alpha')^2\hat{g}_{22}g_{33}g_{tt} = 0 \tag{25b}$$

After substituting induced metric components and $B$ (23) in (25a), one obtains

$$u_*^3(1-\theta^3u_*^3) - (1+\theta^3u_*^3)^2\hat{E}^2 - u_T^3 = 0, \tag{26}$$

whose roots are [6]

$$u_*^3 = \frac{1-2\theta^3\hat{E}^2 \pm \sqrt{1-8\theta^3\hat{E}^2 - 4\theta^3u_T^3 - 4\theta^6u_T^3\hat{E}^2}}{2\theta^3(1+\theta^3\hat{E}^2)}, \tag{27}$$

and by using (25b) conductivity becomes [7]

$$\sigma_{11}^2 = (2\pi\alpha')^4\mathcal{N}^2(\hat{E}^2 + u_T^3)^{2/3}\left(1 + \frac{\theta^3}{3E^2}(3\hat{E} + u_T^3)(5\hat{E} + 3u_T^3)\right), \tag{28}$$

up to $O(\theta^6)$. In the limit of commutative space ($\theta \to 0$), conductivity is in perfect agreement with [9] and it increases in presence of non-commutativity. Two limits corresponding to (7) and (8) are now

- Zero external field

$$\sigma_{11} = (2\pi\alpha')^2\mathcal{N}u_T\sqrt{1 + \frac{\theta^3u_T^6}{E}}, \tag{29}$$

  In commutative case conductivity is proportional to $T^2$ instead of $T$ in (7). Moreover in non-commutative space, correction term plays an important role in high temperature. Note that $\frac{\theta^3u_T^6}{E}$ must always be smaller than 1.

[6] Reality condition of $u_*$ imposes $\hat{E}^2 \leq \frac{1-4\theta^3u_T^3}{4\theta^3(2+\theta^3u_T^3)}$.

[7] For small value of $\theta$, (27) becomes

$$u_*^3 = \frac{1}{2\theta^3}\left(1 - 2\theta^3\hat{E}^2 \pm (1 - 4\theta^3\hat{E}^2 - 2\theta^3u_T^2) + O(\theta^6)\right)$$

where minus sign reproduces commutative solution at non-zero temperature.

- Zero temperature

$$\sigma_{11} = (2\pi\alpha')^2 \mathcal{N} \hat{E}^{2/3} \sqrt{1 + 5(2\pi\alpha')^2\theta^3}. \tag{30}$$

Comparing to $E^{1/2}$ in (8), $E^{2/3}$ appears in conductivity equation in this case.

It is evident that all equations in this subsection reduce to pervious subsection equations by setting $u_T$ to zero.

# 4 A more general non-commutative background

Here we consider a more general non-commutative background given by

$$\begin{aligned} ds^2 =& u^{3/2}\left(h(u)\left(f(u)dt_E^2 + dx_1^2\right) + h'(u)\left(dx_2^2 + dx_3^2\right) + dx_4^2\right) \\ &+ u^{-3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \\ h(u) =& \frac{1}{1+\theta^3 u^3}, \qquad h'(u) = \frac{1}{1+\theta'^3 u^3}, \qquad e^\phi = g_s u^{3/4}\sqrt{h(u)h'(u)} \\ B =& B_{t1} = \theta^{3/2} u^3 h(u), \qquad B' = B_{23} = \theta'^{3/2} u^3 h'(u) \end{aligned} \tag{31}$$

As before, a component of gauge field living on D8-branes is turned on [8]

$$A_1(t_E, u) = iEt_E + a_1(u). \tag{33}$$

The DBI action for $N_f$ D8-branes in background (31) becomes

$$\begin{aligned} S_{D8} = -\mathcal{N}\int dudt_E \frac{u^{1/4}}{\sqrt{hh'}} \\ \times \sqrt{(g_{22}g_{33} + B'^2)\left[g_{tt}(\hat{a}_1'^2 + g_{11}g_{uu}) - g_{uu}(B^2 + \hat{E}^2)\right]}\ , \end{aligned} \tag{34}$$

One can easily simplify the factor $g_{22}g_{33} + B'^2$ appearing in the action which leads to

$$h'(u)u^3, \tag{35}$$

[8] One can introduce a magnetic field on the D8-brane as

$$\mathbb{B}^i = \epsilon^{ijk}\left(B_{jk} + (2\pi\alpha')F_{jk}\right) \tag{32}$$

where in this case we have $E \parallel \mathbb{B}$ and $B \parallel \mathbb{B}$.

and by substituting (35) in the DBI action (34), we have

$$S_{D8} = -\mathcal{N} \int du dt_E \frac{u^{1/4}}{\sqrt{h}} \sqrt{u^3 \big[g_{tt}(\hat{a}_1'^2 + g_{11} g_{uu}) - g_{uu}(B^2 + \hat{E}^2)\big]} \,, \tag{36}$$

Note that $u^{1/4}\sqrt{u^3}$ is exactly the same as $\sqrt{\hat{g}_{22} g_{33}}$ in the action (13) and it is consequently evident that the action (34) and (13) are alike. This result shows that the effect of non-commutativity in "2" and "3" directions can not be recognized by making ansatz (33). In other words non-commutativity does not change the action and consequently the conductivity. The independence of the DBI action on the non-commutativity parameter has been also observed in [14, 16].

We now try another ansatz which is [9]

$$A_3(t_E, u) = iEt_E + a_3(u), \tag{37}$$

and DBI action then becomes

$$S_{D8} = -\mathcal{N} \int du dt_E \frac{u^{1/4}}{\sqrt{hh'}} \times \sqrt{g_{uu}\big[(g_{22}g_{33} + B'^2)(g_{tt}g_{11} + B^2) - g_{11}g_{22}\hat{E}^2\big] + g_{22}\hat{a}_3'^2(g_{tt}g_{11} + B^2)} \,. \tag{38}$$

The conserved charge associated to $a_3(u)$ is given by

$$I_3 \equiv \frac{1}{\sqrt{hh'}} \times \frac{\tilde{g}_{22}(g_{tt}g_{11} + B^2)\hat{a}_3'}{\sqrt{g_{uu}\big[(g_{22}g_{33} + B'^2)(g_{tt}g_{11} + B^2) - g_{11}g_{22}\hat{E}^2\big] + g_{22}\hat{a}_3'^2(g_{tt}g_{11} + B^2)}}, \tag{39}$$

where $\tilde{g}_{22} = u^{1/4} g_{22}$. From (39), it is easy to find $a_3(u)$ which is

$$\hat{a}_3' = \pm \sqrt{\frac{hh' g_{uu}\big[(g_{22}g_{33} + B'^2)(g_{tt}g_{11} + B^2) - g_{11}g_{22}\hat{E}^2\big]}{(g_{tt}g_{11} + B^2)(g_{22}hh' I_3^2 - (g_{tt}g_{11} + B^2)\tilde{g}_{22}^2)}} \; I_3. \tag{40}$$

[9]Due to the rotation symmetry of the background in 23-plane, the other ansatz $A_2(t_E, u) = iEt_E + a_2(u)$ will have the same result as (37). Moreover by using (32) we have $E \perp \mathbb{B}$ and $B \parallel E$.

Conductivity equations coming from (40) are

$$(g_{22}g_{33} + B'^2)(g_{tt}g_{11} + B^2) - g_{11}g_{22}\hat{E}^2 = 0 \tag{41a}$$

$$g_{22}hh'I_3^2 - (g_{tt}g_{11} + B^2)\tilde{g}_{22}^2 = 0 \tag{41b}$$

(31) and (41a) lead to

$$(1 + \theta^3 u_*^3)(u_*^3 - \hat{E}^2) - u_T^3 = 0 \tag{42}$$

In the zero temperature case (*i.e.* $u_T = 0$) non-commutativity, neither $\theta$ nor $\theta'$, does not affect the conductivity and it is like the commutative case [9]. But, in the thermal case we have

$$u_*^3 = \frac{-1 + \theta^3\hat{E}^2 \pm \sqrt{1 + 2\theta^3\hat{E}^2 + \theta^6\hat{E}^4 + 4\theta^3 u_T^3}}{2\theta^3}, \tag{43}$$

and conductivity becomes[10]

$$\sigma_{33}^2 = (2\pi\alpha')^4\mathcal{N}^2(\hat{E}^2 + u_T^3)^{2/3}(1 - \frac{2}{3}\theta^3 u_T^3) \tag{44}$$

up to $O(\theta^6)$. (44) indicates that $\theta'$ does not affect conductivity but $\theta$ decreases value of conductivity. Comparing (28) and (44), it is clearly seen that non-commutativity parameter appears with opposite sign in these two equations depending on which gauge field component is turned on. Although in commutative background there is no difference among $x_1, x_2$ and $x_3$ and therefore conductivity is the same for them *i.e.* $\sigma_{11} = \sigma_{22} = \sigma_{33}$, in noncommutative background we have $\sigma_{11} \neq \sigma_{33} (= \sigma_{22})$.

# 5 Conclusion

The effect of non-commutativity on conductivity was studied in this paper. We considered low and high temperature non-commutative Sakai-Sugimoto and $N_f$ D8-branes were then embedded in these backgrounds. Various components of gauge fields living on D8-branes lead to different values of conductivity in non-commutative background. Our gauge field configurations do not recognize non-commutativity in "2" and "3" directions but non-commutativity in "1" and "2" directions changes the value of conductivity. Although conductivities in different directions, *i.e.* $x^i$, in commutative background are the same, they are not alike in non-commutative background.

[10] Similar to what was discussed in pervious sections, here only the plus sign is acceptable.

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